

# A Simple Consumption-Based Asset Pricing Model and the Cross-Section of Equity Returns\*

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## Abstract

Firm-level risk exposures and costs of equity are notoriously difficult to estimate. Using a novel mapping between consumption risk exposures and firm characteristics, we combine the traditional portfolio-level approach to testing asset pricing models with firm-level information to measure firm-level risk exposures. First, at the portfolio level, we investigate the empirical performance of a simple two-factor consumption-based asset pricing model for the cross-section of equity returns. The priced factors in the model are innovations in the growth and volatility of aggregate consumption. Our empirical results show that this model can explain 78% of the cross-sectional variation in returns on a menu of 55 portfolios spanning size, value, momentum, asset growth, stock issuance, and accruals. Second, we use the estimated model to map point-in-time firm characteristics to consumption risk exposures. Measured in this way, we uncover sizeable cross-sectional and time-series variation in firm consumption risk exposures and are able to predict future firm equity returns.

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# 1 Introduction

Firm-level risk exposures and costs of equity are notoriously difficult to estimate. As emphasized by Fama and French (1997), the challenge is primarily two-fold. First, firm-level risk exposures may be difficult to measure in the face of significant idiosyncratic volatility. This measurement problem is only made worse by the fact that such risk exposures are likely also time-varying. In the face of this challenge, asset pricing researchers have typically employed a portfolio-level analysis to partially alleviate the detection problem (see Black, Jensen and Scholes (1972) and Fama and MacBeth (1973)). This approach has resulted in several decades of research surrounding tests of various asset pricing models, but has offered little insight into solving the problem of measuring firm-level risk exposures.<sup>1</sup> Second, despite the prevalence of the traditional portfolio approach, there remains considerable debate surrounding the identification of the relevant sources of systematic risk in the first place. That is, the appropriate asset pricing model for describing costs of capital has been elusive. Using a novel mapping between consumption risk exposures motivated by recent developments in the asset pricing literature and measurable firm characteristics known to be correlated with expected returns, we combine the traditional portfolio-level approach to testing asset pricing models with firm-level information to uncover firm-level risk exposures.

First, at the portfolio-level, we investigate the empirical performance of a simple two-factor consumption-based asset pricing model for the cross-section of equity returns. The idea that financial asset risk premia reflect compensation for risks inherent in aggregate consumption growth is at the core of financial economics, as shown in the models of Lucas (1978) and Breeden (1979). However, researchers have only relatively recently adapted the consumption-based asset pricing paradigm to successfully model aggregate macroeconomic and asset market moments. In particular, Campbell and Cochrane (1999) use external habit formation to derive a model in which time-varying risk aversion drives asset price dynamics, while Bansal and Yaron (2004) rely on recursive preferences and persistent conditional moments of consumption growth.<sup>2</sup> While these models are broadly successful in capturing macroeconomic and asset pricing moments, it is important to note that they rely on mechanisms and parameterizations that remain hotly debated among financial economists.<sup>3</sup> This debate is unquestionably important for understanding the mechanisms driving

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<sup>1</sup>While asset pricing tests traditionally employ portfolios as test assets, there is a lingering debate about whether firms or portfolios actually serve as the best test assets (see, for example, Ang, Liu, and Schwarz (2010) for a recent contribution). While we focus on the estimation of firm-level risk exposures, we do not directly contribute to this debate.

<sup>2</sup>Habit formation is formally developed in Constantinides (1990), who models internal habit formation and Abel (1990), who models external habit formation. Implications of recursive preferences for asset prices are formally investigated in Epstein and Zin (1989) and Weil (1989).

<sup>3</sup>For example, Mehra and Prescott (2003) question the magnitude and volatility of risk aversion needed for the Campbell and Cochrane (1999) habit formation model to match equity moments. The long run risks model of Bansal and Yaron (2004), in contrast, requires a parameter of intertemporal elasticity of substitution above 1.0, far above the estimate of approximately 0.1 in Hall (1988). Further, the models generate counterfactual implications about other

consumption-based asset pricing models; however, it obscures what we believe to be the most important result of these models, that risks in aggregate consumption have important information for explaining asset prices. An evolving asset pricing literature has undeniably provided mounting evidence that consumption risk exposures can go a long way in explaining cross-sectional variation in average returns on portfolios formed on a broad set of characteristics.

With this background, our goal in this paper is to convince readers that the exposures of asset returns to consumption risks can help improve the characterization of firm-level risk exposures. We do so by first deriving and estimating a reduced form consumption-based asset pricing model that is consistent with structural models of asset prices. Because the model is reduced form, it does not impose any explicit restrictions on preference parameters, and very little structure on consumption dynamics. While investigating the model in reduced form forces us to be silent about the magnitude of preference parameters and the dynamics of consumption growth, it focuses the empirical results on the relation between covariances of asset returns with consumption risks and the cross-section of risk premia.

The paper contributes to a growing list of papers that show that macroeconomic risks, in general, and consumption risks, in particular, are priced in the cross-section. Lettau and Ludvigson (2001) show that a conditional consumption CAPM with a measure of the consumption-wealth ratio as the conditioning variable can explain the cross-section of size- and book-to-market-sorted portfolio returns. Parker and Julliard (2005) demonstrate that the covariance of returns with future consumption growth explains returns on the same set of assets. Bansal, Dittmar, and Lundblad (2005) estimate consumption risk as the covariance of innovations in consumption growth with dividend growth, and show that the resulting risk measures explain size-, book-to-market-, and past 12-month return-sorted portfolios. The role of consumption of durable goods is investigated in Yogo (2006), who finds that adding a measure of durable consumption growth helps the basic consumption model explain size- and book-to-market-sorted portfolio returns. Jagannathan and Wang (2007) conjecture that growth in year- over-year fourth quarter consumption explains expected returns better than simple consumption growth, and find that exposure to this source of risk explains size- and book-to-market-sorted portfolio returns. Hansen, Heaton, and Li (2008) and Bansal, Dittmar, and Kiku (2009) demonstrate that cointegration of cash flows with consumption capture cross-sectional variation in size- and book-to-market-sorted portfolios.<sup>4</sup>

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macroeconomic and asset market moments. The habit formation model counterfactually predicts that consumption growth will predict price-dividend ratios, as pointed out in Bansal, Kiku, and Yaron (2012). In contrast, Beeler and Campbell (2012) point out that moments of consumption growth do not appear to be as persistent as needed in the long run risks framework and that price-dividend ratios counterfactually predict future consumption growth.

<sup>4</sup>In addition to these papers, a number of other papers link macroeconomic risks to the cross-section of equity returns. Lustig and Niewerburgh (2005) examine a conditional C-CAPM with the ratio of housing to human wealth as the conditioning variable. Savov (2011) uses garbage as a proxy for consumption in estimating a C-CAPM. Bansal, Kiku, Shaliastovich, and Yaron (2013) explore the role of macroeconomic volatility in explaining the cross-section of equity returns.

We depart from the aforementioned papers in a number of important dimensions. First, our consumption measure comprises only nondurable goods and services, the standard measure of consumption since, at least, Hansen and Singleton (1982). While we believe that alternative measures of consumption provide additional insights, our results suggest that one can explain a considerable amount of the cross-sectional variation in average returns with a simple measure of aggregate consumption.<sup>5</sup> Second, our measure of risk is the simple covariance of returns with innovations in consumption risk, encompassing not only risk in cash flows but also in capital appreciation. Third, we expand our attention to a broader asset menu than the size- and book-to-market-sorted portfolios that are the focus of most of the above papers. This contribution is important in two dimensions. First, Lewellen, Nagel, and Shanken (2010) show that portfolios sorted on size and book-to-market are easily priced by a two- (or more) factor model. The authors suggest expanding the asset menu to strengthen the empirical conclusions for asset pricing models. Additionally, as shown in Fama and French (2008) and Lewellen (2013), characteristics beyond size and book-to-market are robust predictors of cross-sectional variation in returns.

A key feature of our model is the role that both growth and volatility innovations play in explaining cross-sectional differences in returns; neither source of risk is by itself adequate for understanding risk premia. As a result, our paper also contributes to a literature that examines the implications of volatility risk for asset prices. Ang, Hodrick, Xing, and Zhang (2006) find that exposures to innovations in the VIX command a negative risk premium. Our results are complementary to theirs, as we expect consumption volatility to be highly correlated with the VIX. Indeed, Boguth and Kuehn (2013) find, in a closely related exercise, that consumption volatility risk bears a 7% annual risk premium in the cross-section of returns when measured using portfolios sorted on the basis of consumption volatility risk exposure. Campbell, Giglio, Polk, and Turley (2013) investigate the role of risks in the volatility of a set of potential state variables for explaining cross-sectional variation in returns. In the context of the ICAPM model in Campbell (1993), the authors show that volatility risk is priced and document the importance of volatility risk in explaining cross-sectional variation in a number of asset returns. Bansal, Kiku, Shaliastovich, and Yaron (2013) show that risk in the realized volatility of industrial production growth also bears a large risk premium. Finally, Tédognap (2013) shows that a measure of consumption volatility similar to that employed in our study has strong explanatory power for value and size premia across the term structure of equity returns.<sup>6</sup>

Finally, we propose what we believe to be a novel approach to uncover firm-level risk exposures.

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<sup>5</sup>Uhlig (2007) notes that with time non separable preferences, any consumption over which agents derive felicity will impact pricing, even if intratemporal utility is separable in components of consumption.

<sup>6</sup>Drechsler and Yaron (2011) also investigate the importance of time-varying consumption in volatility for asset prices in the context of option markets. They find that this volatility is important for understanding the volatility risk premium implicit in option prices.

The portfolio-level results suggest that risk exposures, characteristics, and average returns are intricately entwined. Our empirical results show that this model can explain 78% of the cross-sectional variation in returns on a menu of 55 portfolios spanning size, value, momentum, asset growth, stock issuance, and accruals. We use the estimated consumption-based model as the lens through which we map point-in-time firm characteristics to firm-level consumption risk exposures. Measuring firm-level risk in this way, we uncover sizeable cross-sectional and time-series variation in firm consumption risk exposures and are able to predict future firm equity returns. For instance, we use these firm-level exposures to form 25 portfolios that are ex ante sorted on the portfolio-implied risk exposures. The resulting portfolios generate ex post risk exposures consistent with the ex ante ordering of sorts. Further, these portfolios generate greater dispersion in average returns than the 25 size- and book-to-market-sorted portfolios of Fama and French (1993).<sup>7</sup>

The remainder of the paper is organized as follows. In Section 2, we discuss the estimation of consumption innovation risks and the theoretical framework in which these risks are priced. We estimate risk exposures and analyze cross-sectional regressions of portfolio mean returns on risk measures in Section 3. Section 4 presents an analysis of utilizing portfolio characteristics and risk exposures to capture firm-level risk exposures. We make concluding remarks in Section 5.

## 2 Consumption and Expected Returns

### 2.1 Expected Returns and Consumption Moments

The canonical asset pricing model of Lucas (1978) states that an asset’s price is determined by its conditional covariance with a representative agent’s intertemporal marginal rate of substitution (IMRS),

$$E_t [\exp (m_{t+1} + r_{i,t+1})] = 1 \tag{1}$$

where  $m_{t+1}$  is the log IMRS,  $r_{i,t+1}$  is the log gross return on a risky asset  $i$ , and the price of the asset is normalized to unity. Under the further assumption of conditional joint lognormality of the IMRS and the asset return, we can rewrite equation (1) as

$$E_t [r_{i,t+1}] + \frac{1}{2} Var_t (r_{i,t+1}) = -E_t [m_{t+1}] - \frac{1}{2} Var_t (m_{t+1}) - Cov_t (m_{t+1}, r_{i,t+1}). \tag{2}$$

Equation (2) emphasizes the fact that expected returns on assets in the cross-section are related to the covariation of innovations in the IMRS and the asset payoff.

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<sup>7</sup>These results are also related to the evidence that long run risk models perform poorly out of sample in Ferson, Nallareddy, and Xie (2013). While our results do not directly address their concerns, the rolling estimation results in less in-sample dependence than our full-sample results.

A large number of formulations for investors' utility yield a form for the IMRS that is log-linear in the moments of consumption growth. Two cases are of particular interest for our study. The first is power utility, in which the log pricing kernel

$$m_{t+1} = \ln \delta - \gamma \Delta c_{t+1},$$

with  $\gamma$  representing the agent's relative risk aversion,  $\Delta c_{t+1}$  representing log growth in consumption, and  $\delta$  reflecting the agent's time preference. The second is Epstein and Zin (1989) utility, in which the log pricing kernel is represented as

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}.$$

In this expression,  $\psi$  represents the intertemporal elasticity of substitution, which is separable from risk aversion,  $\gamma$ ,  $\theta = (1 - \gamma) / (1 - 1/\psi)$ , and  $r_{c,t+1}$  is the log payoff of an asset that pays aggregate consumption as its dividend. Power utility is a special case where  $\gamma = 1/\psi$  and, consequently,  $\theta = 1$ .

Bansal and Yaron (2004) suggest parameterizing the log return on the consumption claim as a linear function of the state variables of the economy and consumption growth,

$$r_{c,t+1} \approx \kappa_0 + \kappa_1 \mu_{t+1} + \kappa_2 \sigma_{c,t+1}^2 + \Delta c_{t+1}, \quad (3)$$

where  $\mu_{t+1}$  is the conditional expectation of future consumption growth and  $\sigma_{c,t+1}^2$  is its conditional variance. We further assume that

$$\begin{aligned} \Delta c_{t+1} &= \mu_t + \sigma_t \eta_{t+1} \\ \mu_{t+1} &= \mu_c + \rho \mu_t + \varphi \sigma_t \eta_{t+1} \\ \sigma_{t+1}^2 &= E_t [\sigma_{t+1}^2] + \sigma_w w_{t+1}, \end{aligned}$$

where  $\eta_{t+1}$  and  $w_{t+1}$  are standard normal i.i.d. shocks. These dynamics are similar to those explored in Bansal and Yaron (2004), but we assume that the shock to consumption and its conditional mean are the same. As a result, consumption growth is an ARMA(1,1) dynamic process with time-varying volatility.

Under the assumption of log-linearity of the return on the consumption claim in the two state variables, the risk premium on an asset can be determined from equation (2) by

$$\begin{aligned} E [r_{i,t+1} - r_{f,t}] &= -Cov (m_{t+1} - E_t [m_{t+1}], r_{i,t+1} - E_t [r_{i,t+1}]) - \frac{1}{2} Var (r_{i,t+1}) \\ &= \pi_1 Cov (\sigma_t \eta_{t+1}, \eta_{i,t+1}) + \pi_2 Cov (w_{t+1}, \eta_{i,t+1}) - \frac{1}{2} Var (r_{i,t+1}), \end{aligned} \quad (4)$$

where

$$\begin{aligned}\pi_1 &= \frac{\theta}{\psi} - (\theta - 1)(\kappa_1\varphi - 1) \\ \pi_2 &= \kappa_2(\theta - 1),\end{aligned}$$

and  $\eta_{i,t+1} = r_{i,t+1} - E_t[r_{i,t+1}]$ , the shock to the asset return. This expression indicates that investors expect risk premia to compensate for shocks to first moment of consumption risk,  $\eta_{t+1}$ , and second moment of consumption risk  $w_{t+1}$ . Bansal and Yaron (2000) note that under power utility,  $\theta = 1$ , and therefore second moment risk will not be compensated in returns.

Converting back to arithmetic returns, the risk premium (4) can be expressed as

$$E[R_{i,t+1} - R_{f,t}] = \lambda_1\beta_{i,\eta} + \lambda_2\beta_{i,w}, \quad (5)$$

where  $\beta_{i,\eta} = Cov(r_{i,t+1}, \eta_{t+1}) / Var(\eta_{t+1})$  and  $\beta_{i,w} = Cov(r_{i,t+1}, w_{t+1}) / Var(w_{t+1})$  are coefficients of regressing returns on the innovations  $\eta_{t+1}$  and  $w_{t+1}$ . This expression suggests that cross-sectional variation in risk premia will be determined by assets' return exposures to shocks to the first and second moments of consumption growth. Under power utility,  $\lambda_2 = 0$  and only the conditional covariance of consumption growth levels with innovations in asset returns will bear risk premia. Under the further assumption of i.i.d. consumption growth,  $\beta_{i,\eta}$  can be more simply measured by regressing returns on consumption growth.

We close this section by noting that the expression of equation (5) is isomorphic to the risk premium expression in Bansal and Yaron (2004). However, we do not consider the model that we investigate to be a model of long run risk. The reason is that in their model, the magnitude of the covariances of asset returns and risk premia are functions of the dynamics of consumption growth. Specifically, the magnitude of risk exposures and prices of risk are functions of the persistence of the conditional mean of consumption growth,  $\rho$ , and the conditional volatility of consumption growth. Because our approach is reduced form, we are simply allowing the data to inform us as to the magnitude of these risk exposures and prices of risk. While assumptions about dynamics and preference parameters generate powerful predictions about the relations between consumption dynamics, preferences, and asset returns, we focus on the simpler expression of equation (5) in order to highlight sources of cross-sectional variation in risk premia.

## 2.2 Consumption Growth Dynamics

The sources of risk in the model above are the innovations in the level of consumption growth and the volatility of consumption growth. Measuring these innovations depends on modeling the dynamics

of consumption growth. The specification of these dynamics is controversial. The controversy largely stems from the question of whether the conditional mean of consumption growth is constant or not. Working (1960), shows that if a higher frequency i.i.d. process is aggregated to a lower frequency, that the resulting low-frequency process can appear to have positive autocorrelation. As a result, if consumption choices are made at the monthly frequency and are i.i.d., but are measured quarterly or annually, the measured series will appear to have a time-varying conditional expectation. Beeler and Campbell (2012) examine consumption growth at the annual and the quarterly frequency and conclude that the degree of autocorrelation implied by variance ratio tests are close to those implied by the results of Working (1960). In contrast, relying on the standard errors of an estimated autoregression, Piazzesi (2001, p. 321) states that “consumption growth is definitely not i.i.d.” We treat this issue as an empirical question for which there are conflicting pieces of evidence and simply model the level of consumption growth as an i.i.d. process. Our intention is to show that empirically one can generate a lot of cross-sectional variation in predicted expected returns even with the simplest model of mean growth dynamics.<sup>8</sup>

Time variation in the second moment of consumption is somewhat less controversial. Kandel and Stambaugh (1990) document time variation in second moments of consumption growth that appears to be related to the business cycle. Specifically, second moments appear to be high during recessions and relatively low during expansions. We model second moments using an EGARCH(1,1) model of consumption innovations:

$$\Delta c_{t+1} = \mu + \sigma_t \eta_{t+1} \tag{6}$$

$$\ln \sigma_{t+1}^2 = \nu_0 + \nu_1 \ln \sigma_t^2 + \xi \eta_t + \nu (|\eta_t| - E[|\eta_t|]), \tag{7}$$

where  $\eta_t \sim \mathcal{N}(0, 1)$ . We estimate the parameters of the time series model using data on aggregate consumption of nondurable goods and services. The data are obtained from the National Income and Product Accounts at the Bureau of Economic Analysis. We construct the real per capita consumption series as the sum of nondurable goods and services consumption, deflated by the mid period population and the personal consumption expenditures (PCE) deflator. Data are sampled at the quarterly frequency and cover the time period March, 1947 through December, 2012.

Parameter estimates are presented in Table 1. The point estimates suggest that volatility is persistent and exhibits some weak evidence of downside asymmetry. The GARCH parameter of 0.75 is statistically significant (SE=0.08) and indicates positive autocorrelation in volatility. The asymmetry parameter is negative as typically expected, indicating greater downside than upside

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<sup>8</sup>We have also investigated the implications of either AR(1) or ARMA(1,1) dynamics for consumption growth for consumption innovations and the relation of asset return covariances with these innovations and the cross-section of expected returns. Our results are not sensitive to the specification of the dynamics; we find strong cross-sectional explanatory power for the model with all three dynamic specifications.



volatility. The parameter estimate, -0.13, is just over two standard errors from zero (SE=0.06). We use these dynamics to extract innovations in volatility for estimation of risk measures.

### 3 Cross-Sectional Analysis

#### 3.1 Data and Summary Statistics

Asset pricing models have been tested on a wide variety of assets, but arguably the most popular in cross-sectional analysis are sets of equity portfolios. Equity returns have the advantage of having a long time span, which is of particular importance in testing models based on macroeconomic aggregates. The most widely used cross-section is the set of 25 equity portfolios sorted on past market value and book-to-market ratio, first studied in Fama and French (1993). While these portfolios are popular due to their strong cross-sectional variation in average returns, the practice of using these portfolios in cross-sectional regressions has recently been criticized in Lewellen, Nagel, and Shanken (2010). The issue at hand is that the portfolios have a strong three- (or even two-) factor structure, and so models with two or three cross-sectional explanatory variables can generate high regression  $R^2$  even if the variables are only modestly correlated with true sources of cross-sectional variation in returns.

In addition to the above-mentioned concerns, in the years since the publication of Fama and French (1992), a large number of additional variables have been found to have cross-sectional power for explaining variation in average returns beyond that explained by market value and book-to-market ratios. The set of these variables is very large; Lewellen (2013) considers the predictive power of 15 variables, and as noted in Harvey, Liu, and Zhu (2013), some 186 variables have been identified as potential factors for cross-sectional variation in returns. We concentrate on six variables that seem to have robust predictive power for returns; size, book-to-market, past 12-month return, asset growth, total accruals, and stock issuance over the past three years. We construct value-weighted portfolio returns based on deciles of size, book-to-market ratio, past 12-month return, asset growth, and total accruals using data obtained from Compustat and CRSP. We form quintile portfolios based on past stock issuance due to the fact that stock issuance tends to be concentrated in the tails; most firms repurchase, neither issue nor repurchase, or issue equity. In attempting to form decile portfolios, we find that there are months in which we cannot generate ten deciles due to the mass of non-issuing firms; hence the decision to form quintile portfolios. Details on data construction and references to empirical studies documenting the predictive power of these variables are provided in the Appendix.

Summary statistics for the portfolio returns are presented in Table 2. The data cover the

period December, 1953 through December, 2012. All portfolios are value-weighted, and data are sampled at the quarterly frequency and deflated to real using the PCE deflator. Mean returns exhibit patterns that are now familiar to readers of the empirical asset pricing literature; average returns increase in the book-to-market ratio, and past 12-month return, and decrease in market value, asset growth, total accruals, and stock issues. None of the average returns are perfectly monotonic in their characteristic deciles, but some characteristics appear to generate more nearly monotonic patterns than others. In particular, past 12-month returns appear to generate very nearly monotonic patterns in average returns, with only one deviation in the deciles; similarly, stock issuance quintiles deviate in monotonicity only in the middle quintile. The data suggest quite a large dispersion in average returns as well; the highest average return is on the tenth decile past 12-month return portfolio of 4.25%, and the lowest is on the first decile past 12-month return portfolio of -0.68%. The remaining sorts generate differences in returns of 1.22% for the difference in the bottom and top asset growth decile to 1.73% for the difference in the bottom and top market value decile.

### 3.2 Risk Exposures in the Cross Section

We estimate time series regressions of portfolio returns on the innovations in consumption, and its volatility,  $\eta_{t+1}$  and  $w_{t+1}$ ,

$$R_{i,t+1} = a_{i,\eta} + \beta_{i,\eta}\hat{\eta}_{t+1} + \beta_{i,w}\hat{w}_{t+1} + e_{i,t+1}.$$

We report exposures to innovations in the mean of consumption,  $\beta_{i,\eta}$  and associated standard errors in Table 3. The exposures for market value-sorted portfolios exhibit a perfectly monotonically decreasing pattern in market value deciles, which matches the pattern in average returns. The first decile market value portfolio has the largest exposure of all assets in our cross section, and the tenth decile has nearly the smallest. This evidence suggests that much of the size effect can be captured by cross-sectional dispersion in consumption growth innovation exposures; that is, a simple consumption CAPM explains most of the cross-sectional variation in the size-sorted portfolios.

While the simple consumption-based model risk exposures match the average returns of size-sorted portfolios, patterns in the betas of other portfolios are less clear. Examining extreme deciles of book-to-market-sorted and past 12-month return-sorted portfolios suggests a positive relation between growth innovation exposure and these characteristics, which is consistent with patterns in average returns. Further, first decile or quintile asset growth, total accruals, and equity issuance portfolio risk exposures are all larger than the top decile or quintile risk exposures. Again, this pattern is consistent with differences in extreme quantile average returns. However, within quantiles exposures are far from monotonic; patterns in exposures are generally generally U-shaped for

portfolios sorted on characteristics other than market value. All parameters are estimated with precision; with the exception of the bottom decile past 12-month return-sorted portfolio all point estimates are more than two standard errors from zero.

The general impression that we take from these results is that the growth risk exposures have some ability to capture cross-sectional variation in average returns. Extreme quantile risk exposures map reasonably well into extreme quantile average returns. It is less clear, however, that within quantiles the growth risk exposures correlate strongly with within-quantile average returns. Additionally, it is unclear whether the risk exposures correlate well across characteristic sorts with patterns in average returns. These questions are analyzed formally in regressions in the next section.

Exposures to volatility innovations and associated standard errors are presented in Table 4. In this table, market value deciles again show a strong association with volatility innovation risk exposures, falling nearly monotonically across deciles. However, while the evidence on growth risk exposures discussed above is consistent with both patterns in average returns and the predicted sign of the relation, the volatility exposures generate a pattern opposite of that predicted by economic intuition. A high exposure to volatility risk suggests that an asset pays off when volatility innovations are high, times associated with economic downturns. Since we expect marginal utility to be high in these states of the world, high volatility exposure assets are desirable since they insure against bad marginal utility states and should command low risk premia. In the case of the size-sorted portfolios, the opposite pattern appears to hold. Small firms have large volatility innovation risk exposures and high returns, while large firms have low volatility innovation risk exposures and low returns. Thus, the univariate pattern of volatility exposures in market value-sorted returns appears counterintuitive relative to economic theory.

Like the growth risk exposures, sorts on the characteristics other than market value generally produce U-shaped patterns in the exposure of returns to volatility innovations. The lowest volatility risk exposures for book-to-market-, past 12-month return-, asset growth-, and total actuals-sorted portfolios are observed in intermediate deciles. Since these intermediate decile portfolios do not have the highest average returns for the characteristic sorts, the results appear counter to economic intuition. In the case of past 12-month-sorted portfolio returns, the first decile portfolio has a much larger exposure than the tenth decile portfolio, which is consistent with the pattern of tenth decile portfolio returns exceeding first decile portfolio returns on average. However, extreme quantile differences for the remaining sorts are nearly zero. Again, parameters are estimated with reasonable precision; we would fail to reject the null hypothesis that the parameter estimate was different than zero at the 5% significance level using a  $t$ -test for 15 of the 55 portfolios.

One final result that deserves additional discussion is the positive estimates for all portfolio exposures to volatility innovation risk. We were somewhat surprised by this result as it suggests

that all assets covary positively with volatility innovations; that is, all assets tend to have relatively high payoffs when volatility innovations are high. This result is somewhat counterintuitive given our interpretation of equity returns as risky and periods with high volatility innovations as low marginal utility states of the world. The positive exposures can be better understood considering the high correlation between growth innovation risk exposures and volatility innovation risk exposures (0.62). In the long run risks model of Bansal and Yaron (2004), volatility innovation risk exposure is positively correlated with mean innovation risk exposure; the volatility innovation risk exposure is a function of the squared mean innovation risk exposure. The correlation of our risk measures suggests that our results are consistent with their risk exposures. In fact, we find that if we orthogonalize our estimated volatility innovation risk exposures relative to the growth innovation risk exposures, the resulting volatility innovation exposures are all negative. Hence, controlling for their growth innovation risk, the equity portfolios do provide a poor hedge against bad economic states.

### 3.3 Cross-Sectional Regression Results

The standard approach to investigating whether risk exposures are related to average returns is the two-stage approach where returns are regressed on sources of risk and average returns are then regressed on the resulting risk exposure estimates. The first stage estimates are discussed in the previous section, we now examine cross-sectional regressions of the form

$$\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_\eta \hat{\beta}_{i,\eta} + \gamma_w \hat{\beta}_{i,w} + u_i, \quad (8)$$

where  $\bar{R}_i$  is the time series average of the return on portfolio  $i$ ,  $\bar{R}_f$  is the mean real quarterly compounded return on a Treasury Bill closest to one month to maturity from CRSP, and  $\hat{\beta}_{i,\eta}$  and  $\hat{\beta}_{i,w}$  are first stage estimates of univariate regressions of portfolio  $i$ 's return on the mean and volatility innovations,  $\eta_t$  and  $w_t$ , respectively. In addition to the unrestricted model above, we also examine specifications where we consider the explanatory power of mean innovation and volatility innovation risks alone, restricting  $\gamma_\eta = 0$  and  $\gamma_w = 0$ , respectively.

Results of the cross-sectional regressions are presented in Table 5. For each of the three specifications, we present estimates of the intercept and slope coefficients, standard errors of the estimates, and adjusted regression  $R^2$ . The standard errors are corrected for estimation error in the first stage using the correction derived by Shanken (1992). In addition, we present in parentheses under the  $R^2$  the 95% critical value of the model  $R^2$  under the null that the risk measures are unrelated to the average returns. This critical value is motivated by the recommendations of Lewellen, Nagel, and Shanken (2010), who suggest that the cross-sectional  $R^2$  may overstate the model fit. The critical value is calculated by generating 5000 random samples with 263 time series observations of two

normally distributed variables with mean zero and standard deviation  $\sigma_\eta$  and  $\sigma_w$  to match sample standard deviations of the mean and volatility innovations. We regress returns on our sample assets on the random variables, and then perform second stage regressions of the mean returns on the resulting regression coefficients. Adjusted  $R^2$  for the second stage regressions on the simulated risk measures are used to construct the null distribution of the adjusted  $R^2$ .

The first two rows of the table present parameter estimates and standard errors for the model with only mean innovation risk exposures priced,  $\gamma_\eta = 0$ . Consistent with our discussion in the previous section, there is evidence of a univariate relation between average returns and mean innovation risk exposures. The point estimate, 0.344, suggests that mean innovation risk exposures are positively related to average returns, which is consistent with the predictions of a consumption-based asset pricing model, and the point estimate is statistically significant at over three standard errors from zero. The model fares surprisingly well considering the well-documented poor performance of the simple consumption CAPM with an adjusted  $R^2$  of 28.04%. While this adjusted  $R^2$  does not exceed the 95% critical value implied in simulation, it does exceed the 90% critical value. Finally, the null hypothesis that the intercept is equal to zero cannot be rejected. In summary, exposures to consumption growth innovations appear to have significant explanatory power for cross-sectional variation in returns.

In the next two rows of the table, we present the specification in which only volatility innovation risk is priced,  $\gamma_\eta = 0$ . In the previous section, we found limited evidence to suggest that volatility innovation was related to average returns in a univariate sense, with risk measures weakly decreasing in portfolio sorts with increasing average returns and increasing in sorts with decreasing average returns. The regression point estimate of -0.019 is consistent with this intuition, but the estimate cannot be statistically distinguished from zero. Moreover, the regression result suggests that volatility risk exposure by itself has virtually no explanatory power for average returns; the adjusted  $R^2$  is -0.71%. As a result, there is limited evidence that volatility innovation risks exhibit explanatory power for average returns independent of growth innovation exposures.

We last turn to the unrestricted model in the final two rows of the table. The multiple regression results indicate a starkly different conclusion than the univariate results discussed previously. Mean innovation risk exposure is positively priced with a statistically significant coefficient of 0.799 that is more than five standard errors from zero. Volatility innovation risk exposure is also statistically significantly priced; the point estimate of -0.181 is also more than five standard errors from zero. Thus, the point estimates suggest prices of risk that are consistent with economic intuition from consumption-based models; investors demand a reward for assets that covary more with mean innovation exposures and pay a discount for assets that covary more with volatility innovation exposures. Beyond the point estimates, the explanatory power of the regression is also striking; the adjusted  $R^2$  of 78.18 suggests that the model explains more than three quarters of the cross-

sectional variation in average returns. In contrast to the univariate results, this  $\bar{R}^2$  is unlikely to be due to chance as the 95% critical value for the regression  $R^2$  is 48.15. A last point in favor of model performance is the fact that the intercept term cannot be statistically distinguished from zero.

Further corroboration of the model performance is presented in Figure 1, where we plot actual average returns against those predicted by the multiple regression. With an  $R^2$  of one, the point estimates will align themselves on the 45 degree line depicted in the figure. As shown, the majority of the asset returns are clustered along the 45 degree line. The figure also suggests some dimensions along which the model fails to capture all cross-sectional variation in asset returns. High past 12-month return portfolios earn higher returns than predicted by the model and low past 12-month return portfolios earn lower returns than predicted, suggesting that more extreme regression slopes are needed to capture this particular cross-section. In contrast, high asset growth portfolios earn higher average returns than predicted and low asset growth portfolios earn lower average returns than predicted, suggesting that more moderate regression slopes are needed to better capture this cross section of returns. However, the figure suggests that generally the model fares quite well in capturing the cross-sectional variation in average returns.

### 3.4 Alternative Models

#### 3.4.1 Consumption-Based Pricing Models

As mentioned earlier, the past decade or so has witnessed an explosion in consumption-based models that have explanatory power for the cross-section of average returns. The model that we explore in this paper differs from these earlier models in using innovations in the moments of a standard measure of consumption growth as the source of priced risk and measuring risk exposures as the simple covariance between equity returns and these innovations. In contrast, alternative models rely on conditioning information, alternative measures of consumption, or alternative measures of payoffs. In order to gauge the relative performance of this paper's model, we analyze several consumption-based alternatives.

The first alternative that we consider is a conditional consumption CAPM from Lettau and Ludvigson (2001). The authors propose using a measure of the consumption-wealth ratio as a conditioning variable,  $cay_t$ . The conditioning variable is the cointegrating residual from the trivariate cointegrating relation between per capita aggregate consumption, asset wealth, and labor income (measuring the dividend to human wealth). Following their example, we estimate the following

two-stage cross-sectional regression:

$$R_{i,t+1} - R_f = a_i + \beta_{i,cay}cay_t + \beta_{i,\Delta c}\Delta c_{t+1} + \beta_{i,cay \cdot \Delta c}cay_t \cdot \Delta c_{t+1} + e_{i,t+1} \quad (9)$$

$$\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{cay}\beta_{i,cay} + \gamma_{\Delta c}\beta_{i,\Delta c} + \gamma_{cay \cdot \Delta c}\beta_{i,cay \cdot \Delta c} + u_i, \quad (10)$$

where  $\Delta c_{t+1}$  is the growth in per capita consumption of nondurables and services. Data for  $cay_t$  are obtained from Martin Lettau's web page.<sup>9</sup>

The second alternative is an unconditional consumption CAPM using a measure of *ultimate* consumption examined in Parker and Julliard (2005). Ultimate consumption growth is defined as the  $s$ -period forward growth in consumption of nondurables and services,

$$g_{t+1,t+s+1} = \sum_{j=0}^s \Delta c_{t+j+1}$$

where, following the authors' evidence, we set  $s = 11$ . We consider the case of their log-linearized model with a constant risk-free rate,

$$R_{i,t+1} - R_f = a_i + \beta_{i,g}g_{t+1,t+s+1} + e_{i,t+1} \quad (11)$$

$$\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_g\beta_{i,g} + u_i. \quad (12)$$

Consumption data are the same as those used earlier in the paper; however, due to the horizon  $s$ , the return data are truncated in 2009.

A third alternative is investigated in Bansal, Dittmar, and Lundblad (2005), who suggest that the covariance of portfolio cash flows with a measure of the conditional mean of consumption growth explains cross-sectional variation in returns. Their measure of the conditional mean is a moving average of consumption growth,

$$x_t = \frac{1}{K} \sum_{j=0}^{K-1} \Delta c_{t-j},$$

where the authors set  $K = 8$ . The model investigated is specified as

$$\Delta d_{i,t+1} = a_i + \beta_{i,x}x_t + e_{i,t+1} \quad (13)$$

$$\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_x\beta_{i,x} + u_i \quad (14)$$

where  $\Delta d_{i,t+1}$  is the log growth in real dividends per share paid by portfolio  $i$ . Portfolio dividends

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<sup>9</sup><http://faculty.haas.berkeley.edu/lettau/data.html>. Thanks to Martin Lettau for making these data available.

per share are constructed through the recursion

$$\begin{aligned} V_{i,t} &= V_{i,t-1} (1 + R_{i,t}^x) \\ D_{i,t} &= V_{i,t-1} (R_{i,t} - R_{i,t}^x), \end{aligned}$$

where  $R_{i,t}^x$  is the ex-dividend portfolio return,  $D_{i,t}$  is the arithmetic dividend per share and  $V_0 = 100$ . Seasonalities are removed from dividends by summing over twelve months.

The final consumption-based alternative that we consider is the durable consumption model of Yogo (2006). In his framework, preferences are non-separable in consumption of nondurables, services, and durable goods. A log-linear approximation to the model results in the following specification:

$$R_{i,t+1} - R_{f,t} = a_i + \beta_{i,nds} \Delta c_{nds,t+1} + \beta_{i,d} \Delta c_{d,t+1} + \beta_{i,m} R_{m,t+1} + e_{i,t+1} \quad (15)$$

$$\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{nds} \beta_{i,nds} + \gamma_d \beta_{i,d} + \gamma_m \beta_{i,m} + u_i, \quad (16)$$

where  $\Delta c_{nds,t+1}$  is the growth in log real per capita nondurable and services consumption,  $\Delta c_{nd,t+1}$  is growth in consumption of durable goods, and  $R_{m,t+1}$  is the return on the value-weighted market. The durable goods consumption data are taken from Motohiro Yogo's website.<sup>10</sup> These data are available through December, 2001.

The results of estimation of the four consumption-based alternative models are shown in Table 6. All four of the models fare reasonably well in terms of generating statistically significant coefficients on their hypothesized sources of priced risk. Ultimate consumption growth (point estimate=3.001, SE=1.347), consumption growth (point estimate=0.466, SE=0.113), durable consumption growth (point estimate=0.959, SE=0.332), and conditional mean of consumption growth (point estimate=0.166, SE=0.032) all bear positive prices of risk with point estimates more than two standard errors from zero. In terms of adjusted  $R^2$  the four models display more heterogeneity in performance; the ultimate consumption model explains the least cross-sectional variation in returns with an adjusted  $R^2$  of 13.82%, while the durable consumption model explains 52.72% of cross-sectional variation in returns. All four models perform better in terms of adjusted  $R^2$  on a similar set of assets as those on which they were originally tested. On the size- and book-to-market portfolios, the conditional CCAPM, ultimate consumption model, and durable goods models explain 65.73%, 66.00%, and 54.12% of cross-sectional variation in returns respectively. The cash flow conditional consumption mean model explains 54.04% of the cross-sectional variation in returns sorted on size-, book-to-market, and past 12-month returns.

The conclusion that we draw from these results is that a model based on risks in innovations

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<sup>10</sup><https://sites.google.com/site/motohiroyogo/>. Thanks to Moto Yogo for making these data available.



in the growth and volatility of nondurables and services consumption goes very far in explaining cross-sectional variation in returns. While the use of conditioning information, ultimate consumption growth, cash flows, and durable goods are all potentially important in understanding cross-sectional variation in returns, the moment innovations model dominates these models in terms of cross-sectional explanatory power.<sup>11</sup> Our interpretation is that a model with innovations in growth and volatility of nondurables and services consumption represents a strong starting point for understanding cross-sectional variation in returns.

### 3.4.2 Return Factor Models

Our last analysis in this section is the performance of models with return-based rather than consumption-based sources of risk. In particular, we examine two return factor models, the Sharpe (1964), Lintner (1965), Black (1972) CAPM, and the Fama and French (1993) three-factor model. The former uses the market return, or equivalently market risk premium, the return on a broad-based portfolio of equities in excess of the risk-free rate as a single factor for explaining returns. The latter augments this market portfolio with hedge portfolios formed on the basis of the difference in returns on a low market capitalization and high market capitalization portfolio and the difference in returns on a portfolio of firms with high book to market equity ratios and a portfolio of firms with low book to market equity ratios. The Fama and French (1993) model is inspired by the evidence in Fama and French (1992) that size and book-to-market are variables that subsume all others examined in explaining cross-sectional variation in returns. We obtain data on the market risk premium (*MRP*), market capitalization (*SMB*), and book-to-market (*HML*) return factors from Kenneth French’s website.<sup>12</sup> We aggregate returns on six size and book-to-market portfolios, the market portfolio, and the risk-free rate to the quarterly frequency, and deflate the returns by the PCE deflator. We use the resulting returns to construct quarterly observations on *MRP*, *SMB*, and *HML*.

As above, we examine two stage regressions where the first stage consists of univariate regressions of returns on the 55 portfolios in our sample on each of the three return risk factors. In the second stage, we regress average returns in excess of the risk free rate on the risk exposures from the second stage. We again use the Shanken (1992) correction to compute standard errors and construct the distribution of adjusted  $R^2$ . We examine the CAPM and Fama-French models above, and then conduct a “horse race” in which we augment the CAPM and Fama-French risk exposures by the growth and volatility innovation risk exposures examined previously.

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<sup>11</sup>When we estimate the model over a common time sample as the durable goods model, ending in 2001, the model adjusted  $R^2$  exceeds 67%.

<sup>12</sup>[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). Our thanks to Kenneth French for making these data available.

Results of these regressions are presented in Table 7. Consistent with much of the existing empirical evidence, the CAPM fares very poorly in describing cross-sectional variation in returns. While the price of risk is more than two standard errors from zero, it is negative in contradiction with the prediction of the CAPM which suggests that market beta bears a positive price of risk. The model explains very little of the cross-sectional variation in returns, with an adjusted  $R^2$  of 6.27. This does not exceed the 95% critical value of 42.30, suggesting that the adjusted  $R^2$  cannot be distinguished from that achieved by random exposures.

The Fama-French model fares better, but also has some difficulties in explaining cross-sectional variation in this set of returns. The price of market and size risk are statistically distinguishable from zero, but the price of book-to-market-risk is not. Moreover, two of the three prices of risk are negative. As in the case of the CAPM, beta risk appears to bear a negative price, even when controlling for the effects of risks embodied in the size and book-to-market factors. Additionally, the book-to-market factor has a negative price of risk, which is problematic since the motivation for the book-to-market factor is that it is a factor that loads positively on high book-to-market, which are assumed to be high risk. The cross-sectional explanatory power of the model is reasonable with an adjusted  $R^2$  of 47.44. However, the 95% critical value for a three factor model with mean and variance of the three factors is 63.64, suggesting that the model's performance in describing cross-sectional variation cannot be distinguished from chance.

In the last two sets of regression results in the table, we present regressions of average returns on the factor risk exposures augmented by the mean and volatility risk exposures estimated earlier. As shown in the table, the estimates prices of growth and volatility innovation risks are little changed from those in Table 5. Growth innovation risks remain positively and significantly priced in both specifications, and volatility innovation risk is negatively and statistically significantly priced. The market, size size and book-to-market factor risk prices are not statistically significant different than zero in either of these two specifications. Finally, the regression adjusted  $R^2$  are not substantially higher than those with the growth and volatility innovation risks alone in Table 5. However, both adjusted  $R^2$  are above the 95% critical threshold of the distribution under the null.

### 3.5 Volatility and the Cross-Section of Returns

Our cross-sectional results suggest that neither growth rate nor volatility effects in consumption are sufficient to capture risks that describe cross-sectional variation in equity risk premia. Rather, it is the interaction of these risks that is important for understanding the required rate of return on equities. As discussed in the Introduction, we are not the first paper to investigate the role of volatility risk in explaining cross-sectional variation in returns. Recent investigations into this source of risk include Ang, Hodrick, Xing, and Zhang (2006), who examine the role of innovations

in the VIX in explaining cross-sectional variation in returns, Bansal, Kiku, Shaliastovich, and Yaron (2013), who focus on news about the realized variation in industrial production growth, and Campbell, Giglio, Polk, and Turley (2013), who investigate the role of news about shocks to rolling market volatility. In this section, we compare the properties and pricing implications of innovations to consumption growth volatility with the sources of risk examined in their frameworks.

There are some significant differences in our EGARCH(1,1) measures of volatility innovations and alternative implementations. In both Bansal, Kiku, Shaliastovich, and Yaron (2013) and Campbell, Giglio, Polk, and Turley (2013), the priced risks are news shocks, or impulse responses from a VAR of several state variables, to measures of volatility. In contrast, the measure of risk in our approach is the current shock to volatility based on a particular volatility model. To provide a level comparison, we consider an approach to modeling volatility based on an EGARCH(1,1) model applied to the sources of risk in these two papers; the growth in industrial production and the value-weighted market return.<sup>13</sup> The VIX data are only available starting in 1986; over a common time period, we find that the EGARCH(1,1) consumption volatility is 27% correlated with the VIX, and virtually uncorrelated with innovations in the VIX utilized in Ang, Hodrick, Xing, and Zhang (2006).

In Figure 2, we plot EGARCH(1,1) volatility of consumption growth, the market return, and industrial production growth. Volatilities are standardized to zero mean and unit standard deviation to facilitate comparison. NBER recessions are shaded in grey. As shown in the figure, all three volatility measures are positively correlated, and have a tendency to rise in recessions and fall in expansions. The measures of macroeconomic volatility have a reasonably high correlation; volatility in consumption growth and volatility in industrial production growth have a correlation coefficient of approximately 53%. These measures are, however, less correlated with the measure of market volatility. Consumption and industrial production growth volatilities are 14% and 24% correlated with the market return volatility, respectively. Some of this difference is observed due to episodic market volatility that is not reflected in volatility in real activity. For example, large spikes are observed during the 1987 market crash, the Asian currency crisis, the September 11, 2001 attacks, and the Flash Crash that are not reflected in the macroeconomic measures of volatility.

While the correlations of the industrial production-implied volatility with consumption-implied volatilities are modest, the risk measures estimated using industrial production-implied volatility are highly correlated with consumption-volatility risk exposures. As in equation (8), we estimate

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<sup>13</sup>We do compare our consumption volatility obtained from an EGARCH(1,1) model to quarterly observations on realized volatility in industrial production growth and rolling volatility of the return on the CRSP value-weighted index. Our measure of volatility is 35% correlated with realized volatility in industrial production growth and 19% correlated with rolling volatility in the market return. Our innovations in consumption volatility are virtually uncorrelated with the news shocks related to industrial production realized volatility and rolling volatility in the market return.

multiple regressions of returns on the innovation in consumption growth and the alternative volatility innovation measures,

$$\begin{aligned} R_{i,t+1} &= a_{i,ip} + \beta_{i,\eta,ip}\hat{\eta}_{t+1} + \beta_{i,w_{ip}}\hat{w}_{ip,t+1} + e_{i,ip,t+1} \\ R_{i,t+1} &= a_{i,rm} + \beta_{i,\eta,rm}\hat{\eta}_{t+1} + \beta_{i,w_{rm}}\hat{w}_{rm,t+1} + e_{i,rm,t+1}, \end{aligned}$$

where  $w_{ip,t+1}$  and  $w_{rm,t+1}$  are the innovations to EGARCH(1,1) volatility of industrial production growth and the return on the market, respectively. The consumption volatility innovation risk measures are 89% correlated with industrial production volatility innovation risk measures. However, like the volatilities, risk exposures to consumption and industrial production volatility innovations are less correlated with the market volatility innovation exposures. Consumption volatility exposures are 15% correlated with market volatility exposures, and -1% correlated with industrial production volatility exposures. Thus, the exposures suggest that macroeconomic and market-based volatility innovations are capturing different risks.

Despite the differences in the risk exposures between the macroeconomic volatility measures and the market volatility measure, all three sets of risk exposures fare reasonably well in explaining the cross-section of average returns when paired with consumption growth innovation exposures. When we estimate the cross-sectional regression in equation (8), we find that replacing the consumption volatility innovation risk exposures with industrial production volatility innovation risk exposures yields a statistically significant negative coefficient and an adjusted  $R^2$  of 71.59. The model with market volatility innovation risk exposures also produces a negative and statistically significant coefficient, and an adjusted  $R^2$  of 55.81. While the explanatory power of these models is slightly below that of the consumption-based model, the results suggest some common information in risk exposures to aggregate volatility innovations. Most importantly, the results underscore the importance of including measures of aggregate volatility exposure in understanding variation in the cross-section return on equities.<sup>14</sup>

## 4 Firm Characteristics and Risk Exposures

The evidence in Section 3 suggests that exposures of asset returns to innovations in the mean and volatility of consumption growth go a long way in explaining cross-sectional variation in average returns on a set of assets formed on a broad set of characteristics. The fact that this model works well to explain portfolio-level average returns suggests that measured firm-level risk exposures could also explain firm-level average returns. Unfortunately, measuring firm-level exposures to macroeconomic

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<sup>14</sup>A full set of tabulated results with EGARCH(1,1) estimates, risk exposures, and cross-sectional regressions are available from the authors upon request.

risks are compounded by a number of issues. Macroeconomic risks are associated with economic contractions, of which there are relatively few in the data. As a result, the econometrician needs a long time sample to estimate these risks. For many firms, especially small firms and initial public offerings, a sufficiently long time span of data may not be available. Further, while portfolio-level risk exposures might be relatively constant, it is far less clear that firm-level risk exposures are constant. This consideration further compounds the issue of the long data span needed to estimate macroeconomic risk exposures. Finally, the relative low frequency of macroeconomic data relative to equity return data is yet another limiting factor in estimating firm-level macroeconomic risks.

In this section, we propose what we believe to be a novel approach to assessing firm-level exposures to economic risk. The results of the previous section suggest that risk exposures, characteristics, and average returns are intricately entwined. We suggest that the link is due to the fact that the characteristics are instruments for risk exposures. This idea can be traced at least to Fama and French (1992), who suggest that the reason that size and book-to-market command cross-sectional risk premia is because they represent exposures to unknown risk factors. In contrast to their setting, we observe measures of the risk exposures at the portfolio level, and use this information to infer risk exposures at the firm level through characteristics.

#### 4.1 Characteristics and Risk Exposures in the Cross Section

Our motivation for using firm characteristics as measures of risk exposure draws on the production-based asset pricing literature, which suggests that characteristics related to the firm’s capital structure and production technology are related to return covariances with the stochastic discount factor. In particular, Lin and Zhang (2013) make explicit the link between firm risk exposures and characteristics embedded in production technologies. In order to investigate the link between these two quantities, we examine portfolio-level cross-sectional regressions:

$$\hat{\beta}_{i,\eta} = a_{i,\eta} + \mathbf{b}'_{i,\eta} \bar{\mathbf{x}}_i + e_{i,\eta} \quad (17)$$

$$\hat{\beta}_{i,w} = a_{i,w} + \mathbf{b}'_{i,w} \bar{\mathbf{x}}_i + e_{i,w}, \quad (18)$$

where  $\bar{\mathbf{x}}_i$  is a vector of the time series average of portfolio  $i$ ’s characteristics; size, book-to-market ratio, past 12-month return, asset growth, total accruals, and stock issuance. We construct portfolio characteristics by value-weighting the individual characteristics of each asset in the portfolio over 714 months from July, 1953 through December, 2012.

Regression results are shown in Table 8. The first set of results are for regressions of the growth innovation risk exposure on the average characteristics. The results indicate that four variables are significantly related to the growth risk exposure; book-to-market, size, past 12-month return,

and total accruals. Book-to-market, size, and total accruals are negatively related to the exposures, while past 12-month returns are positively related. These signs are consistent with the prices of risk from the earlier results with the exception of the book-to-market ratio; in general, characteristics associated with higher returns are associated with higher risk exposures. Approximately 79% of the variation in the risk measures can be captured through the average characteristics.

The second set of results in Table 8 show regression results from regressing volatility innovation betas on the characteristics. In this case, three of the six characteristics are at least marginally statistically significantly different than zero; book-to-market ratio, size, and past 12-month return. All three of these variables are negatively related to volatility innovation risk exposures. As with the growth innovation risk exposures, two of the three signs are consistent with volatility innovation risk premia. High book-to-market and high past 12-month return firms are associated with high returns and low volatility innovation risk exposures. The market value sign suggests that large firms have lower exposures, which contrasts with their lower average returns. 75% of the variation in volatility risk measures is associated with the average characteristics for the portfolios.

These results suggest that characteristics may indeed instrument for risk exposures. As noted above, our particular interest is in whether the relation between the characteristics and the risk exposures can be extended to the firm growth. We next turn to addressing this question using individual firm data.

## 4.2 Characteristics and Firm Risk Exposures

The evidence presented in the previous section suggests that at the portfolio level, characteristics and risk measures are highly correlated. Our interpretation of these results, consistent with production-based models as in Lin and Zhang (2013), is that characteristics instrument for risk exposures. Furthermore, as argued in Lin and Zhang (2013), because covariances are estimated with error, characteristics can appear to dominate true covariances in explaining cross-sectional variation in returns. Consequently, characteristics instrument for and can be used as proxies for risk exposures.

We assume that the portfolio-level relation between risk exposures and characteristics holds at the firm level. That is, the relation between firm-level exposures and portfolio-level exposures and

characteristics is given by

$$\begin{aligned}\beta_{i,e,t} &= a_t + \mathbf{b}'_t \mathbf{x}_{it} + u_{it} \\ \beta_{p,e,t} &= \sum_{i=1}^N \omega_{i,t-1} (a_t + \mathbf{b}'_t \mathbf{x}_{it} + u_{it}) \\ &= a_t + \mathbf{b}'_t \mathbf{x}_{pt} + u_{pt},\end{aligned}$$

where  $i$  indexes firms,  $p$  represents portfolios,  $\omega_i$  is the weight on asset  $i$  in the portfolio, and  $e = \eta, w$ . Consequently, by estimating the coefficients  $a_t$  and  $\mathbf{b}_t$  at the portfolio level, we can use the coefficients to retrieve firm-specific risk measures at the firm level. This translation between the portfolio and the firm level relations of risk exposures and characteristics is similar to the use of portfolio-level CAPM betas to measure firm-level betas in Fama and French (1992).

The specific procedure by which we apply portfolio-level estimates to the firm level proceeds as follows:

1. Somewhat arbitrarily, we choose a time span  $T = 120$ , or 30 years of data from July, 1953 to June, 1982 to estimate initial risk exposures. We estimate a model of consumption dynamics over this 30 years to obtain the innovations to the level and volatility of consumption growth, and estimate exposures to these innovations by regressing portfolio returns on the innovations. We then regress the resulting risk exposures on the portfolio characteristics,  $\mathbf{X}_{it}$  for each month July, 1982 through September, 1982, and retain the regression parameter estimates. Note that this approach uses only data available at June, 1982 to estimate both consumption dynamics and relations between characteristics and risk exposures.
2. We roll forward one quarter, augmenting the consumption growth and return data by the new quarter's observations, and re-estimate the model of consumption dynamics and accompanying risk exposures. We then regress characteristics for each month October, 1982 through December, 1982 on the risk exposures. Since our characteristics use the timing convention of Fama and French (1993), these characteristics are based on financial statement data known as of June, 1982. We continue this procedure, expanding the window over which the model of consumption dynamics and regression of returns on innovations is estimated until reaching the end of the sample.
3. We take the regression estimates from steps 1 and 2 and estimate betas at the firm level given the firm-level characteristics and the point estimates. We rank firms into quintiles on the basis of the estimated growth innovation beta, and then rank within quintiles on the basis of the implied volatility innovation beta, and form value-weighted portfolios on the basis of these rankings. We do not independently sort on the estimated betas due to their correlation;

in some months there are no intersections of some of the growth and volatility innovation quintiles.

Using the estimated growth and volatility exposures from the above procedure, we construct a set of 25 value-weighted portfolios, sorting into quintiles on the basis of the predicted growth risk exposure and, within each growth risk exposure quintile, into quintiles on the basis of the predicted volatility innovation risk exposure.<sup>15</sup>

Means of the resulting portfolio returns are shown in Panel A of Table 9. Returns are sampled at the monthly frequency over the period July, 1983 through December, 2012. The portfolio sorts produce a very wide range of average returns across portfolios. The highest mean portfolio return is that of the low imputed volatility risk exposure within the fifth quintile of imputed growth risk exposure, at 175 basis points per month, and the lowest mean portfolio return is that of the highest volatility risk exposure quintile within the lowest mean portfolio quintile at 11 basis points per month. This spread in average returns of 164 basis points is extremely large; as a point of comparison, the spread in average monthly returns for the size- and book-to-market portfolios analyzed in Fama and French (1993) over the same period is 122 basis points.<sup>16</sup> Average returns are nearly monotonically increasing across mean and decreasing across volatility quintiles. The average return across the five mean quintiles falls monotonically from 126 basis points per month for the average of first quintile returns to 39 basis points per month for the average of fifth quintile returns. Averaging across volatility quintiles, returns increase from 74 basis points per month for the bottom mean quintile portfolio to 125 basis points per month for the top mean quintile portfolio.

While it is encouraging that the sorting procedure generates significant dispersion in average returns, we wish to know more fundamentally whether the procedure generates predicted dispersion in risk measures. That is, while we speculate that the relation between portfolio risk measures and characteristics extends to the firm level, does this conjecture in fact hold? Moreover, does the *ex ante* prediction of the approach for risk exposures result in portfolios that are *ex post* sorted on exposures to growth and innovation risk? We address this question by estimating risk exposures for the portfolios using regressions specified in equation (8). In order to estimate exposures, we compound portfolio returns to the quarterly frequency and deflate to real using the PCE deflator.

Ex post growth innovation risk exposures are presented in Panel B of Table 9. As shown in the table, ex ante portfolio sorts on imputed growth innovation risk measures perform well in producing ex post growth innovation risk measures. The fifth growth exposure quintile exceeds that of the first quintile for each of the five volatility quintiles. Risk exposures are close to monotonically increasing across each mean quintile as well, with a single departure from monotonicity in each of

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<sup>15</sup>We experimented with sorts on the intersection of quantiles as well. In order to ensure that each intersection had at least one firm for each month over the sample, the sort required tercile rather than quintile sorting.

<sup>16</sup>Data are from [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).



the first, second, and fifth volatility innovation risk quintiles. The other pattern that emerges is an ex post relation between the growth innovation and volatility innovation sorts. In general, growth innovation exposures are decreasing across volatility innovation exposures. Similar results are shown in Panel C of Table 9 for the volatility innovation risk exposure. In each of the mean quintiles, ex post volatility innovation risk exposures are higher for the fifth volatility innovation risk exposure quintile than the first quintile. Again, the patterns are not perfectly monotonic, with a single departure from monotonicity in each of the third, fourth, and fifth growth innovation quintiles. These results suggest that ranking on predicted growth and volatility innovation exposures generally predicts future growth and volatility innovation risk exposures.

## 5 Conclusion

Consumption-based asset pricing is an essential link between standard economic theory and finance. The basic prediction of the theory is that risks in consumption growth are priced in the cross-section of asset returns. We present evidence in this paper that these risks are indeed priced in the cross-section of equity returns. More specifically, firms with relatively higher covariation with innovations in the level of consumption growth and relatively lower covariation with innovations in the volatility of consumption growth command relatively high risk premia. Our framework explains over 75% of the cross-sectional variation in average returns across a broad menu of characteristic-sorted portfolios, more than alternative consumption-based alternatives and return factor-based alternatives.

We also propose a novel approach for using the information in portfolio risk exposures and characteristics to measure firm-level risk exposures. We find that portfolio-level risk exposures are strongly linked to portfolio-level characteristics, and use this relation to generate portfolios sorted on predicted individual firm level and volatility innovation risk exposures. The resulting portfolios generate substantial variation in average returns, with a mean spread of 164 basis points per month over the period July, 1983 through December, 2012. The resulting portfolios also generate spreads in measured exposures to growth and volatility innovation risks, with predicted rankings aligning closely with actual rankings.

The results presented in this paper suggest that a simple consumption-based model serves as a benchmark for asset pricing, dominating even return-based factor models in explaining cross-sectional variation in returns. Our suggestion to researchers is to consider benchmarking against this model for risk due to its broad explanatory power. Characteristics can be used to impute firm-level risk exposures for the purpose of risk adjustment in a wide variety of applications. Further, portfolios sorted on the basis of predicted consumption risk exposures have potential to be a

powerful set of test assets for asset pricing theory.

## Appendix A Construction of Test Portfolios

Our empirical tests utilize portfolios sorted on six characteristics: market value, book-to-market ratio, past 12-month return, asset growth, total accruals, and stock issuance. We describe the construction of these portfolios in this appendix. Accounting variables used to construct the characteristics are obtained from Compustat and data on returns, prices, and shares outstanding are obtained from CRSP. All accounting variables are matched to subsequent returns using the procedure in Fama and French (1993); it is assumed that the accounting variable known from June of year  $t$  through July of year  $t + 1$  is the value as of fiscal year end financial statements ending in calendar year  $t - 1$ .

### *Market Value*

Banz (1981) and Fama and French (1992) document the ability of size to explain cross-sectional variation in returns. Firm size for July of year  $t$  through June of year  $t + 1$  is measured as the CRSP market value of the firm at the end of June of year  $t$ . Market value is the product of CRSP shares outstanding times CRSP price per share.

### *Book to Market*

Fama and French (1992) show that the book-to-market ratio, together with market value dominate many other characteristics in describing cross-sectional variation in returns. The book-to-market ratio at June of year  $t$  is the book value of equity in June divided by the market value of equity at December of year  $t - 1$ . Following Fama and French (2008), we measure book value of equity as book equity plus balance sheet deferred taxes (Compustat item TXDB), investment tax credits (ITCB), and preferred stock. Book equity is total assets (AT) less total liabilities (LT). Preferred stock is, in order of preference, liquidating value (PSTKL), redemption value (PSTKRV), or carrying value (UPSTK).

### *Past 12 Month Return*

Momentum strategies, defined as buying recent past winners and selling recent past losers is shown to be profitable in Jegadeesh and Titman (1993). The profit is maximized when the window over which past returns are calculated is 12 months and the holding period is three months. We calculate cumulative returns for firms over months  $t - 11$  through  $t - 2$  for portfolio formation in month  $t$ . Jegadeesh and Titman (1993) show that skipping one month enhances returns due to avoidance of one-month reversals. Past returns are recalculated each month.

### *Asset Growth*

Cooper, Gulen, and Schill (2008) show that firms with low growth in total assets outperform

firms with high growth in total assets on average. In the same spirit as the authors, we calculate the growth in assets as the difference in log assets at June of year  $t$  and June of year  $t - 1$ . Total assets are Compustat item AT.

#### *Total Accruals*

Firms with low total accruals have higher average returns than firms with high total accruals as shown in Sloan (1996). Accruals are measured as the change in working capital dividend by average total assets. Working capital is measured as current assets (ACT) minus cash and equivalents (CHE) minus current liabilities (LCT) plus debt in current liabilities (DLC). Average total assets is the average of total assets (AT) at June of year  $t$  and June of year  $t - 1$ .

#### *Stock Issues*

Pontiff and Woodgate (2008) show that sorting on net share issuance generates differences in average returns with firms with low net issuance outperforming firms with high net issuance. Following Fama and French (2008), we measure net stock issues as the log difference in split-adjusted CRSP shares outstanding from year  $t - 1$  to year  $t$ .

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Table 1: Consumption Volatility Dynamics

Table 1 presents estimates of an EGARCH(1,1) model for the volatility of aggregate consumption growth dynamics,

$$\begin{aligned}\Delta c_{t+1} &= \mu + \eta_{t+1} \\ \ln \sigma_{t+1}^2 &= \nu_0 + \nu_1 \ln \sigma_t^2 + \xi e_t + v (|e_t| - E[|e_t|]),\end{aligned}$$

where  $\Delta c_{t+1}$  is the growth in log real per capita nondurable goods and services and  $\sigma_{t+1}^2$  is the conditional volatility of consumption growth. The model is estimated via maximum likelihood using quarterly consumption data from March, 1947 through December, 2012. Consumption is deflated to real using the PCE deflator.

	$\nu_1$	$\xi$	$v$
Coeff.	0.751	0.674	-0.128
SE	(0.083)	(0.100)	(0.060)

Table 2: Average Returns

Table 2 depicts average returns on a set of 55 portfolios formed on the basis of six characteristics. Portfolios are formed on market value (MV), book-to-market ratio (BM), past 12-month return (P12), asset growth (AG), total accruals (TA), and net stock issues (SI). We form value-weighted portfolios based on deciles of the first five characteristics and quintiles of net stock issues. Data are sampled at the quarterly frequency from September, 1953 through December, 2012. Returns are deflated to real using the PCE deflator from the NIPA tables at the Bureau of Economic Analysis.

Dec/Quint	MV	BM	P12	AG	TA	SI
1	3.72	1.93	-0.68	2.83	2.87	2.89
2	3.03	1.77	0.44	2.60	2.58	2.26
3	3.07	2.24	0.97	2.48	2.32	2.30
4	2.81	2.00	1.72	2.23	2.05	2.11
5	2.72	2.35	1.94	2.32	2.19	1.36
6	2.90	2.41	1.92	2.19	2.15	
7	2.67	2.45	2.30	2.29	2.13	
8	2.60	2.83	2.67	2.27	2.15	
9	2.54	2.91	2.96	2.15	1.84	
10	1.99	3.49	4.25	1.61	1.39	

Table 3: Consumption Growth Innovation Risk Exposures

In Table 3, we present grown innovation risk exposures and standard errors from a regression of portfolio excess returns on consumption growth level and volatility innovations,

$$R_{i,t+1} - R_{f,t} = a_i + \beta_{i,\eta}\hat{\eta}_{t+1} + \beta_{i,w}\hat{w}_{t+1} + e_{i,t+1},$$

where  $\hat{\eta}_{t+1}$  is the innovation in the level of consumption growth and  $\hat{w}_{t+1}$  is the innovation in consumption volatility from an EGARCH(1,1) model. Returns are on portfolios sorted on market value (MV), book-to-market ratio (BM), past 12-month return (P12), asset growth (AG), total accruals (TA), and net stock issues (SI). We present point estimates of growth risk exposures,  $\beta_{i,\eta}$  in Panel A and standard errors for the estimates in Panel B. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.

Panel A: Estimates

Decile	MV	BM	P12	AG	TA	SI
1	10.43	6.51	7.41	8.11	7.22	5.29
2	9.91	5.44	7.25	5.42	6.50	4.90
3	9.17	5.09	5.62	4.97	4.46	5.92
4	8.93	4.74	5.00	4.03	3.85	5.74
5	8.19	4.73	4.80	4.13	4.41	4.79
6	7.92	4.74	4.63	5.16	4.92	
7	7.27	4.81	4.67	5.23	5.39	
8	6.58	4.83	4.59	5.32	5.58	
9	5.96	4.71	5.69	5.82	6.41	
10	4.82	6.61	7.53	6.41	7.28	

Panel B: Standard Errors

Decile	MV	BM	P12	AG	TA	SI
1	(2.22)	(1.58)	(2.65)	(1.65)	(1.57)	(1.16)
2	(2.02)	(1.35)	(2.04)	(1.39)	(1.43)	(1.17)
3	(1.98)	(1.33)	(1.68)	(1.24)	(1.25)	(1.29)
4	(1.85)	(1.32)	(1.48)	(1.14)	(1.14)	(1.39)
5	(1.81)	(1.22)	(1.34)	(1.15)	(1.13)	(1.44)
6	(1.75)	(1.19)	(1.24)	(1.23)	(1.22)	
7	(1.66)	(1.29)	(1.23)	(1.25)	(1.40)	
8	(1.57)	(1.32)	(1.24)	(1.38)	(1.45)	
9	(1.44)	(1.34)	(1.37)	(1.54)	(1.51)	
10	(1.20)	(1.56)	(1.82)	(1.69)	(1.80)	

Table 4: Volatility Innovation Risk Exposures

In Table 4, we present volatility innovation risk exposures and standard errors from a regression of portfolio excess returns on consumption growth level and volatility innovations,

$$R_{i,t+1} - R_{f,t} = a_i + \beta_{i,\eta}\hat{\eta}_{t+1} + \beta_{i,w}\hat{w}_{t+1} + e_{i,t+1},$$

where  $\hat{\eta}_{t+1}$  is the innovation in the level of consumption growth and  $\hat{w}_{t+1}$  is the innovation in consumption volatility from an EGARCH(1,1) model. Returns are on portfolios sorted on market value (MV), book-to-market ratio (BM), past 12-month return (P12), asset growth (AG), total accruals (TA), and net stock issues (SI). We present point estimates of volatility innovation risk exposures,  $\beta_{i,w}$  in Panel A and standard errors for the estimates in Panel B. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.

Panel A: Estimates

Decile	MV	BM	P12	AG	TA	SI
1	27.36	26.53	47.94	20.05	23.47	9.87
2	25.27	23.13	37.07	9.00	12.30	12.55
3	27.61	17.07	21.36	9.24	8.51	13.71
4	27.87	13.01	18.47	6.92	6.30	13.53
5	25.62	7.60	23.41	9.68	11.95	18.12
6	25.73	6.78	16.47	10.52	13.68	
7	23.28	7.48	15.75	9.89	17.69	
8	22.39	0.18	8.46	18.57	20.15	
9	21.36	11.95	10.88	28.87	26.40	
10	11.86	10.97	14.98	31.38	32.17	

Panel B: Standard Errors

Decile	MV	BM	P12	AG	TA	SI
1	(17.53)	(12.28)	(20.16)	(13.07)	(12.33)	(9.14)
2	(16.02)	(10.48)	(15.69)	(10.85)	(11.31)	(9.13)
3	(15.56)	(10.33)	(13.00)	(9.71)	(9.74)	(10.20)
4	(14.62)	(10.23)	(11.45)	(8.88)	(8.86)	(10.89)
5	(14.19)	(9.56)	(10.31)	(8.95)	(8.84)	(11.14)
6	(13.75)	(9.32)	(9.61)	(9.62)	(9.55)	
7	(13.00)	(10.08)	(9.56)	(9.79)	(10.90)	
8	(12.25)	(10.28)	(9.63)	(10.70)	(11.27)	
9	(11.23)	(10.43)	(10.75)	(11.87)	(11.75)	
10	(9.36)	(12.29)	(14.30)	(13.08)	(13.96)	

Table 5: Cross-Sectional Regressions

Table 5 presents estimates of cross-sectional regressions of average excess portfolio returns on risk measures,

$$\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_\eta \beta_{i,\eta} + \gamma_w \beta_{i,w} + u_i,$$

where  $\bar{R}_i$  is the average real quarterly return on a set of 55 portfolios formed on market value, book-to-market ratio, past 12-month return, asset growth, total accruals, and net stock issues, and  $\bar{R}_f$  is the real quarterly compounded return on a Treasury Bill closest to one month to maturity. The independent variables  $\beta_{i,\eta}$  and  $\beta_{i,w}$  are slope coefficients from a first stage regression,

$$R_{i,t+1} - R_{f,t} = a_{i,\eta} + \beta_{i,\eta} \hat{\eta}_{t+1} + \beta_{i,w} \hat{w}_{t+1} + e_{i,t+1},$$

where  $\hat{\eta}_{t+1}$  is the innovation in the level of consumption growth and  $\hat{w}_{t+1}$  is the innovation in consumption volatility from an EGARCH(1,1) model. The table presents point estimates and adjusted  $R^2$  for versions of the model with the restrictions  $\gamma_w = 0$ ,  $\gamma_\eta = 0$ , and an unrestricted version. Standard errors corrected for first stage estimation bias following Shanken (1992) are presented in parentheses below the point estimates. Beneath the adjusted  $R^2$ , we present 95% critical values for adjusted  $R^2$  from 5000 Monte Carlo simulations under the null that the independent variables have no explanatory power for the returns. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.

	$\gamma_0$	$\gamma_\eta$	$\gamma_w$	$R^2$
Coeff.	0.587	0.344		28.04
SE	(0.392)	(0.089)		(41.47)
Coeff.	2.262		-0.019	-0.71
SE	(0.287)		(0.025)	(43.17)
Coeff.	0.627	0.799	-0.181	78.18
SE	(0.380)	(0.123)	(0.035)	(55.14)

Table 6: Alternative Consumption Models

We investigate the performance of alternative consumption-based pricing models in Table 6. We consider four models. The first is a conditional consumption CCAPM from Lettau and Ludvigson (2001),

$$\begin{aligned} R_{i,t+1} - R_{f,t} &= a_i + \beta_{i,cay} cay_t + \beta_{i,\Delta c} \Delta c_{t+1} + \beta_{i,cay \cdot \Delta c} cay_t \cdot \Delta c_{t+1} + e_{i,t} \\ \bar{R}_i - \bar{R}_f &= \gamma_0 + \gamma_{cay} \beta_{i,cay} + \gamma_{\Delta c} \beta_{i,\Delta c} + \gamma_{cay \cdot \Delta c} \beta_{i,cay \cdot \Delta c} + u_i, \end{aligned}$$

where  $\Delta c_{t+1}$  is log real per capita growth in consumption of nondurables and services and  $cay_t$  is the cointegrating residual from a trivariate cointegrating relation between aggregate consumption, asset wealth, and labor income. The second alternative is the ultimate consumption model of Parker and Julliard (2005),

$$\begin{aligned} R_{i,t+1} - R_{f,t} &= a_i + \beta_{i,g} g_{t+1,t+s+1} + e_{i,t+1} \\ \bar{R}_i - \bar{R}_f &= \gamma_0 + \gamma_g \beta_{i,g} + u_i. \end{aligned}$$

where  $g_{t,t+s}$  is cumulative growth in per capita consumption of nondurables and services over quarters  $t$  through  $t+s$ , with  $s = 11$ . The third model is the consumption cash flow risk model of Bansal, Dittmar, and Lundblad (2005),

$$\begin{aligned} \Delta d_{i,t+1} &= a_i + \beta_{i,x} x_t + e_{i,t+1} \\ \bar{R}_i - \bar{R}_f &= \gamma_0 + \gamma_x \beta_{i,x} + u_i \end{aligned}$$

where  $\Delta d_{i,t+1}$  is the log growth in real dividends per share on portfolio  $i$  and  $x_t$  is the average growth in consumption over quarters  $t-7$  through  $t$ . The final model is the durable consumption model of Yogo (2006),

$$\begin{aligned} R_{i,t+1} - R_{f,t} &= a_i + \beta_{i,nds} \Delta c_{nds,t+1} + \beta_{i,d} \Delta c_{d,t+1} + \beta_{i,m} R_{m,t+1} + e_{i,t+1} \\ \bar{R}_i - \bar{R}_f &= \gamma_0 + \gamma_{nds} \beta_{i,nds} + \gamma_d \beta_{i,d} + \gamma_m \beta_{i,m} + u_i, \end{aligned}$$

where  $\Delta c_{nds,t+1}$  is growth in real per capita consumption of nondurables and services,  $\Delta c_{d,t+1}$  is growth in real per capita consumption of durable goods, and  $R_{m,t+1}$  is the real return on the CRSP value-weighted index. We obtain data for  $cay_t$  and durable consumption growth from Martin Lettau's and Mothohiro Yogo's websites, respectively. Shanken (1992)-corrected standard errors are presented below point estimates in parentheses. Data are sampled quarterly over the period September, 1953 through December, 2012 with the exception of the durable goods model, for which data ends in December, 2001.

Model	Estimates				
<i>Conditional CCAPM</i>	$\gamma_0$	$\gamma_{cay}$	$\gamma_{\Delta c}$	$\gamma_{cay \cdot \Delta c}$	$\bar{R}^2$
Coeff	0.045	0.618	0.466	0.005	44.56
SE	(0.783)	(0.966)	(0.113)	(0.006)	(64.30)
<i>Ultimate Consumption</i>	$\gamma_0$	$\gamma_g$			$\bar{R}^2$
Coeff	0.659	3.001			13.82
SE	(0.603)	(1.347)			(44.29)
<i>Cash Flow Consumption</i>	$\gamma_0$	$\gamma_x$			$\bar{R}^2$
Coeff	1.698	0.166			39.36
SE	(0.102)	(0.032)			(47.72)
<i>Durable Consumption</i>	$\gamma_0$	$\gamma_{nds}$	$\gamma_d$	$\gamma_m$	$\bar{R}^2$
Coeff	2.640	-0.058	0.959	-0.307	52.72
SE	(1.122)	(0.363)	(0.332)	(1.058)	(60.24)

Table 7: Return Factor Models

Table 7 presents results of cross-sectional regressions based on return factor models. Two models are examined, the CAPM, specified as

$$\begin{aligned} R_{i,t+1} - R_{f,t} &= a_i + \beta_{i,m} (R_{m,t+1} - R_{f,t}) + e_{i,t+1} \\ \bar{R}_i - \bar{R}_f &= \gamma_0 + \gamma_{mrp} \beta_{i,m} + u_i, \end{aligned}$$

where  $R_{m,t+1}$  is the CRSP value-weighted market portfolio return, and the Fama and French (1993) three-factor model,

$$\begin{aligned} R_{i,t+1} - R_{f,t} &= a_i + \beta_{i,m} (R_{m,t+1} - R_{f,t}) + \beta_{i,smb} R_{smb,t+1} + \beta_{i,hml} R_{hml,t+1} + e_{i,t+1} \\ \bar{R}_i - \bar{R}_f &= \gamma_0 + \gamma_{mrp} \beta_{i,m} + \gamma_{smb} \beta_{i,smb} + \gamma_{hml} \beta_{i,hml} + u_i, \end{aligned}$$

where  $R_{smb,t+1}$  is the return on the size factor and  $R_{hml,t+1}$  is the return on the value factor. Data for the factor returns are obtained from Kenneth French's website. The table presents point estimates with Shanken (1992)-corrected standard errors in parentheses below the point estimates. We also present adjusted  $R^2$  with 95% critical values from Monte Carlo simulations under the null that the independent variable has no explanatory power for returns. The second and fourth rows of the table augment the CAPM and Fama-French models with consumption innovation risk exposures. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012 and are deflated to real using the PCE deflator from the NIPA tables at the Bureau of Economic Analysis.

	$\gamma_0$	$\gamma_{mrp}$	$\gamma_{smb}$	$\gamma_{hml}$	$\gamma_\eta$	$\gamma_w$	$\bar{R}^2$
Coeff.	3.634	-1.255					6.23
SE	(0.632)	(0.587)					(32.51)
Coeff.	7.812	-7.819	2.452	-1.591			48.00
SE	(1.408)	(1.986)	(0.641)	(0.776)			(57.61)
Coeff.	2.704	-2.624			0.566	-0.055	73.29
SE	(1.150)	(1.721)			(0.109)	(0.026)	(63.33)
Coeff.	3.608	-3.757	0.671	-0.962	0.482	-0.063	75.86
SE	(1.540)	(1.965)	(0.885)	(0.730)	(0.194)	(0.027)	(72.04)

Table 8: Relation Between Characteristics and Risk Measures

Table 8 presents results of regressions of portfolio risk exposures on average portfolio characteristics. Risk exposures,  $\beta_{i,\eta}$  and  $\beta_{i,w}$  are regression coefficients from time series regressions of portfolio returns on innovations in consumption growth and volatility innovations, respectively:

$$R_{i,t+1} = a_i + \beta_{i,\eta}\eta_{t+1} + \beta_{i,w}w_{t+1} + e_{i,t+1}.$$

The characteristics, asset growth (AG), book-to-market ratio (BM), market value (MV), past 12-month return (P12), net stock issues (SI), and total accruals (TA) are constructed at the firm level and value-weighted to calculate portfolio characteristics. The regressions are specified as

$$\begin{aligned}\beta_{i,\eta} &= g_0 + g_1\overline{AG}_i + g_2\overline{BM}_i + g_3\overline{MV}_i + g_4\overline{P12}_i + g_5\overline{SI}_i + g_6\overline{TA}_i + v_i \\ \beta_{i,w} &= h_0 + h_1\overline{AG}_i + h_2\overline{BM}_i + h_3\overline{MV}_i + h_4\overline{P12}_i + h_5\overline{SI}_i + h_6\overline{TA}_i + z_i.\end{aligned}$$

Data are sampled at the monthly frequency over the period September, 1953 through December, 2012.

Dep. Var.		AG	BM	MV	P12	SI	TA	$\overline{R}^2$
$\beta_{i,\eta}$	Coeff.	0.296	-0.399	-0.584	2.509	-0.927	-4.302	79.03
	SE	(0.997)	(0.195)	(0.049)	(0.416)	(1.276)	(2.115)	
$\beta_{i,w}$	Coeff.	5.172	-3.537	-1.570	-7.741	9.010	2.219	74.96
	SE	(3.824)	(0.748)	(0.187)	(1.596)	(4.893)	(8.109)	



Table 9: Innovation Risk-Sorted Portfolios

Table 9 presents mean returns and risk exposures for portfolios formed on predicted risk measures. Predicted risk measures are formed by regressing the risk exposures from expanding window regressions of the returns through time  $t$  on 55 portfolios sorted on six characteristics, asset growth (AG), book-to-market ratio (BM), market value (MV), past 12-month return (P12), net stock issues (SI), and total accruals (TA) on the innovation in the level and volatility of consumption growth on observed characteristics at time  $t$ . The portfolio level coefficients are used to predict risk exposures at the firm level. Portfolios are formed by sorting firms into quintiles on predicted growth innovation risk exposure and then within each quintile into quintiles on the basis of volatility innovation exposure. Panel A presents monthly average portfolio returns over the period July, 1982 through December, 2012. Panels B and C present ex post risk exposures from regressing quarterly portfolio returns on growth and volatility innovation risks over the period July, 1982 through December, 2012.

Panel A: Mean Returns

$\beta_{i,w}$	$\beta_{i,\eta}$ Quintile				
	1	2	3	4	5
1	0.85	1.11	1.22	1.37	1.75
2	0.96	1.13	1.08	1.31	1.50
3	0.96	1.07	1.17	0.82	1.24
4	0.83	0.73	1.03	0.92	0.99
5	0.11	0.36	0.49	0.24	0.76

Panel B: Ex Post  $\beta_{i,\eta}$

$\beta_{i,w}$	$\beta_{i,\eta}$ Quintile				
	1	2	3	4	5
1	3.30	5.28	6.47	6.13	8.78
2	2.33	1.57	3.87	4.77	7.39
3	0.57	1.09	3.38	4.80	6.27
4	-0.80	0.93	3.37	3.50	4.67
5	-0.64	-0.45	5.97	3.38	5.86

Panel C: Ex Post  $\beta_{i,w}$

$\beta_{i,w}$	$\beta_{i,\eta}$ Quintile				
	1	2	3	4	5
1	-1.17	-1.89	2.71	4.18	12.02
2	-0.61	1.17	3.15	4.19	10.14
3	7.06	2.47	1.14	0.37	10.94
4	11.12	2.64	5.00	14.27	15.31
5	24.65	21.27	16.14	17.48	17.72

Figure 1: Actual and Predicted Average Returns

Figure 1 presents a scatterplot of average returns on 55 portfolios sorted on the basis of asset growth (AG), book-to-market ratio (BM), market value (MV), past 12-month return (P12), net stock issues (SI), and total accruals (TA) on predicted returns from the regression

$$\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_\eta \beta_{i,\eta} + \gamma_w \beta_{i,w} + u_i,$$

where  $\beta_{i,\eta}$  and  $\beta_{i,w}$  are slope coefficients from the time series regression

$$R_{i,t+1} - R_{f,t} = a_{i,\eta} + \beta_{i,\eta} \hat{\eta}_{t+1} + \beta_{i,w} \hat{w}_{t+1} + e_{i,t+1},$$

where  $\hat{\eta}_{t+1}$  is the innovation in the level of consumption growth and  $\hat{w}_{t+1}$  is the innovation in consumption volatility from an EGARCH(1,1) model. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.

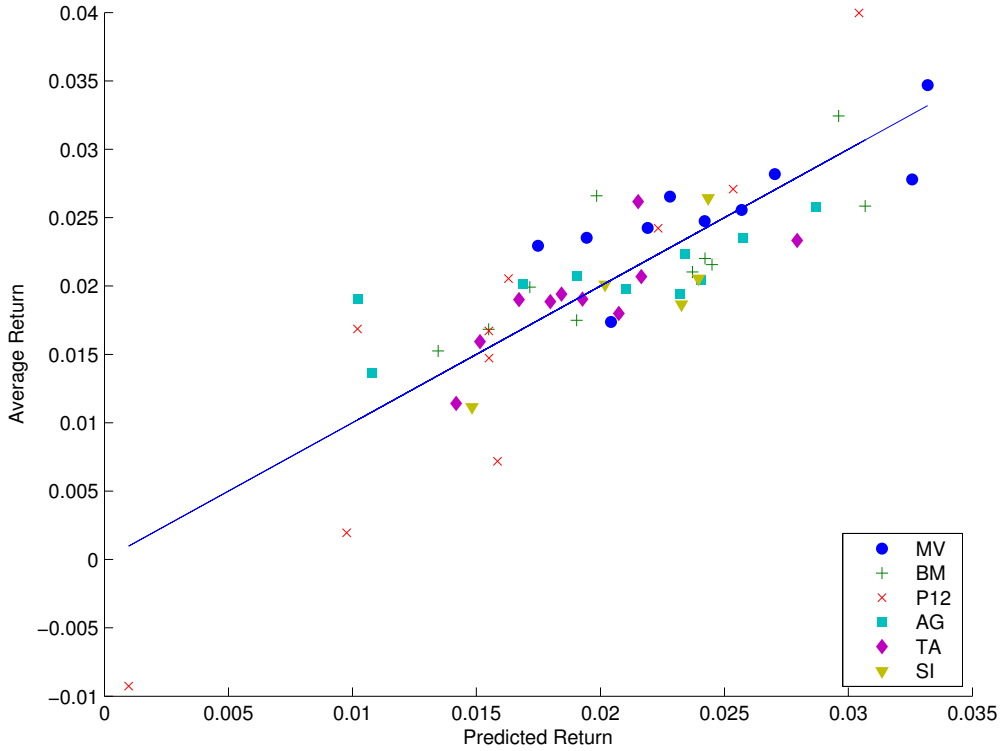


Figure 2: Measures of Aggregate Volatility

Figure 2 presents time series of different measures of aggregate volatility implied by EGARCH(1,1) models. The series examined are log growth in real per capita expenditures of nondurables and services, log growth in industrial production, and the real return on the value-weighted CRSP index. Consumption growth is calculated using information in the NIPA tables from the Bureau of Economic Analysis, industrial production growth is calculated using data from the FRED database at the Federal Reserve Bank of St. Louis, and the CRSP value-weighted return is obtained from CRSP. Data are sampled at the quarterly frequency from June, 1947 through December, 2012, and are deflated to real values using the PCE deflator. Data are standardized to mean zero and unit standard deviation, and NBER recessions are depicted shaded in grey.

